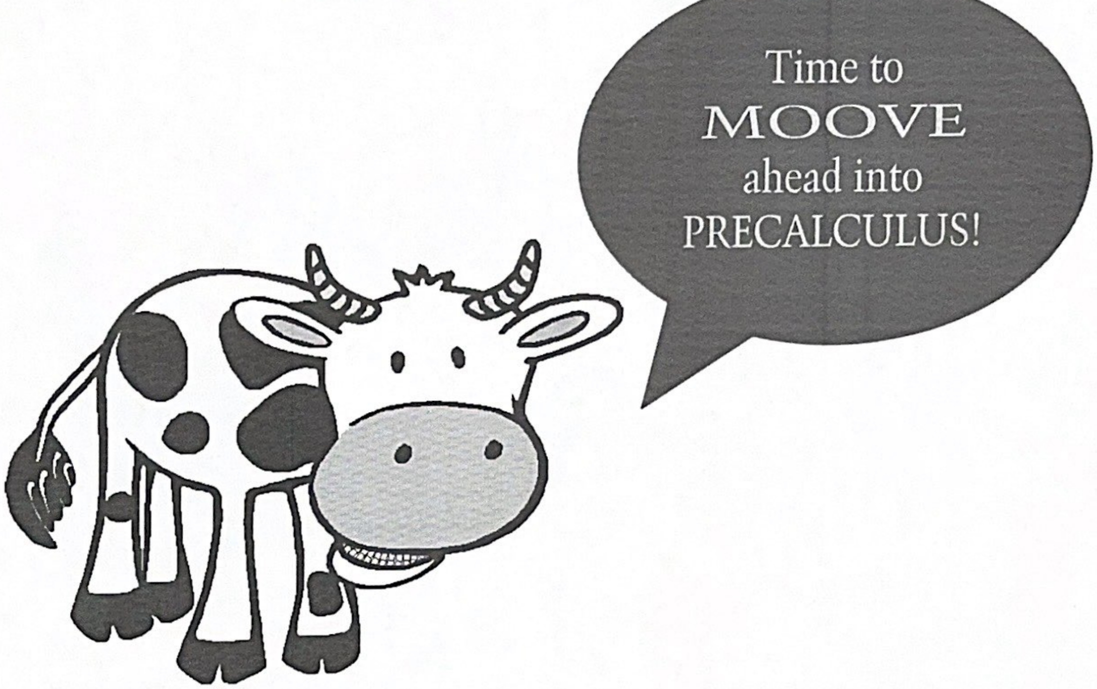
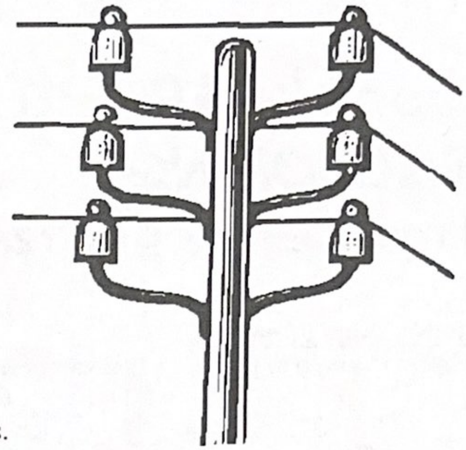


PRECALCULUS SUMMER PREP/ ALGEBRA 2 REVIEW PACKET

SKILL AND CONCEPT REVIEW FOR STUDENTS MOVING FROM ALGEBRA II TO PRECALCULUS

POWERS!

a.k.a. exponents !



Fill in the blanks to complete each rule.

1. When multiplying like bases, _________ the powers.

2. When dividing like bases, ___________ the powers.

3. When raising a power to a power, _____________ the powers.

4. When there is a negative exponent in the ___________________, the term can be moved to the *numerator* and the exponent changed to positive. When there is a negative exponent in the ____________________, the term can be moved to the *denominator* and the exponent changed to positive.

Use the rules of exponents to simplify each term.

1. $x^3y^8 \cdot x^4y^6$

2. $3a^4b^2 \cdot -6a^3b^5$

3. $10x^2yz \cdot 5x^2y^3z^6 \cdot xy$

4. $\frac{x^3y^8}{x^4y^6}$

5. $\frac{3a^4b^2}{-6a^3b^5}$

6. $\frac{10x^2yz}{5x^2y^3}$

7. $(3a^2b^4c^5)^3$

8. $(-7m^6n^3)^2$

9. $2(-2x^2y^3z)^5$

10. $\frac{a^{-3}b^4}{c^6}$

11. $\frac{6x^{-5}y^3}{z^{-8}}$

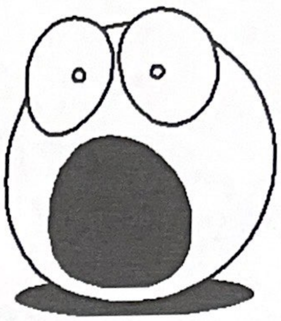
12. $\frac{r^{-5}s^9}{-7s^{-3}t^{-1}}$

UH OH! COMMON ERROR! 

$(a + b^3)^2 = a^2 + b^6$ is FALSE!

Why can't you square each term and apply the power rules listed above?

Don't be FRIGHTENED of FACTORING!



Practice finding patterns!

Recognizing Formats

Select the type of factoring that can be applied by selecting the correct letter.

____ 1. $a^3 + 27$ ____ 2. $9x^2 - 25$ ____ 3. $4a^2b^3 + 6a^3b - 12a^4b^2c$

____ 4. $x^3 + 2x^2 + 3x + 6$ ____ 5. $x^2 + 10x + 16$ ____ 6. $2x^2 + 7x - 15$

a. Difference of squares
b. GCF
c. Trinomial with leading coefficient = 1
d. Trinomial with leading coefficient ≠ 1
e. Sum/difference of cubes
f. Factor by grouping

What should your answers look like?

Below are six different types of factoring and six different examples of possible answer formats. The a's and b's represent integers. Select the answer format that could be matched with each type of factoring.

____ 1. Difference of squares ____ 4. Trinomial with leading coefficient ≠ 1

____ 2. GCF ____ 5. Trinomial with leading coefficient = 1

____ 3. Sum/difference of cubes ____ 6. Factoring by grouping

a. $a(x + b)$
b. $(x + a)(x - a)$
c. $(bx + a)(ax - b)$
d. $(x + a)(x + b)$
e. $(x + a)(x^2 - ax + a^2)$
f. $(a + x^2)(x + b)$

TIP:

You can always multiply out the factored answer to check your work!

True or false?

When factoring $9 - x^2$, the problem should be rewritten with the x^2 first.

FACTORING FRENZY!!!

Factor each expression and match it with its factored form below.

Resist the urge to make your matches "backwards" using multiplication (although multiplication is a good way to *check* your work.)

Note: Some answer choices will not be used. Careful...some may be close to the correct answers, but not quite right!

_____ 1. $8x^3 - 1$ _____ 2. $2x^2 - x$ _____ 3. $x^2 - 3x - 18$

_____ 4. $x^2 - 2x$ _____ 5. $2x^2 + 9x - 5$ _____ 6. $x^2 - 25$

_____ 7. $2x^2 + 5x - 3$ _____ 8. $x^2 - 3x$ _____ 9. $x^3 + 2x^2 - 3x - 6$

_____ 10. $x^3 - 1$ _____ 11. $9 - x^2$ _____ 12. $x^3 + 2x^2$

_____ 13. $x^2 - x - 30$ _____ 14. $x^2 - 9$ _____ 15. $3x + 24 + x^3 + 8x^2$

a. $x(x - 3)$ b. $(2x - 1)(x + 5)$ c. $x^2(x - 2)$ d. $(x + 5)(x - 5)$ e. $x^2(x + 2)$

f. $(2x - 1)(4x^2 + 4x + 1)$ g. $(x - 6)(x + 3)$ h. $(x^2 + 3)(x + 8)$ i. $(x + 5)(x + 5)$

j. $(2x + 1)(x - 5)$ k. $(x - 3)(x - 2)$ l. $(3 - x)(3 + x)$ m. $x(x - 2)$

n. $(x + 3)(x - 3)$ o. $(x - 1)(x^2 + x + 1)$ p. $(2x - 1)(x + 3)$ q. $(x - 6)(x - 3)$

r. $x(2x - 1)$ s. $(2x + 1)(x - 3)$ t. $(x - 6)(x + 5)$ u. $(x - 1)(x^2 - x + 1)$

v. $(2x - 1)(4x^2 + 2x + 1)$ w. $(x^2 - 3)(x + 2)$

Simplifying RADICALS

STARTING WITH THE BASICS

Fill in the blanks for the following statements.

1. To take the square root of a number, the number must be a perfect ________________.

2. To take the square root of a variable, the power must be divisible by __________.

3. To take the cube root of a number, the number must be a perfect ____________.

4. To take the cube root of a variable, the power must be divisible by ____________.

List the squares of each number. The first three have been done for you.

$1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 =$ ________ $5^2 =$ ________

$6^2 =$ ________ $7^2 =$ ________ $8^2 =$ ________ $9^2 =$ ________

$10^2 =$ ________ $11^2 =$ ________ $12^2 =$ ________ $13^2 =$ ________

List the cubes of each number. The first two have been done for you.

$1^3 = 1$ $2^3 = 8$ $3^3 =$ ________ $4^3 =$ ________ $5^3 =$ ________ $6^3 =$ __________

Simplify each radical.

$\sqrt{25} =$ _______ $\sqrt{169} =$ ________ $\sqrt[3]{125} =$ ________ $\sqrt{x^2} =$ _________ $\sqrt[3]{x^3} =$ ________

MOVING BEYOND THE BASICS

Now, carefully READ the following detailed steps for simplifying <u>square roots</u>, and answer the questions along the way. We'll consider how to deal with numbers and variables separately in this process...

STEP ONE – Rewriting the Radical

Numbers:

- *Consider factor pairs of the number (#'s you can multiply together to get the number).* **What are factor pairs of 12?**____________________
- *Select the set that has a perfect square (if your choices contain more than one perfect square, choose the largest).* 12 = ____ · ____
- *Rewrite the number as the product of two numbers.* $\sqrt{12}$ = $\sqrt{\_\_ \cdot \_\_}$
- **Example:** $\sqrt{32} = \sqrt{16 \cdot 2}$ *(4 is also perfect square factor of 32, but 16 is bigger, so use that)*

Variables:

- *If the power isn't divisible by 2… " peel off " a variable so that it is! (that is, subtract the power by one, and multiply that term by the same variable to the 1st power)* $\sqrt{x^{11}} = \sqrt{\_\_ \cdot \_\_}$
- **Example:** $\sqrt{x^7} = \sqrt{x^6 \cdot x}$

STEP TWO – Simplifying the Radical

Numbers:

- *Take the square root of the perfect square and put it in front of your radical (to show multiplication)* $\sqrt{12} = \sqrt{\_\_ \cdot \_\_} = \_\_\_\_\sqrt{\_\_}$
- *If the radical had a number in front of it, multiply the two numbers together.* $5\sqrt{12} = 5\sqrt{\_\_ \cdot \_\_} = \_\_\_\_\sqrt{\_\_}$
- **Example:** $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$
- **Example:** $6\sqrt{75} = 6\sqrt{25 \cdot 3} = 6 \cdot 5\sqrt{3} = 30\sqrt{3}$

Variables:

- *Take your variable that has the even power and divide it by two, and put that answer in front of your radical.* $\sqrt{x^{11}} = \sqrt{\_\_ \cdot \_\_} = \_\_\_\sqrt{\_\_}$
- *If the radical had the same variable in front of it, add the powers.*

  $x^7\sqrt{x^{11}} = x^7\sqrt{\_\_ \cdot \_\_} = \_\_\_\sqrt{\_\_}$ **(examples on next page)**

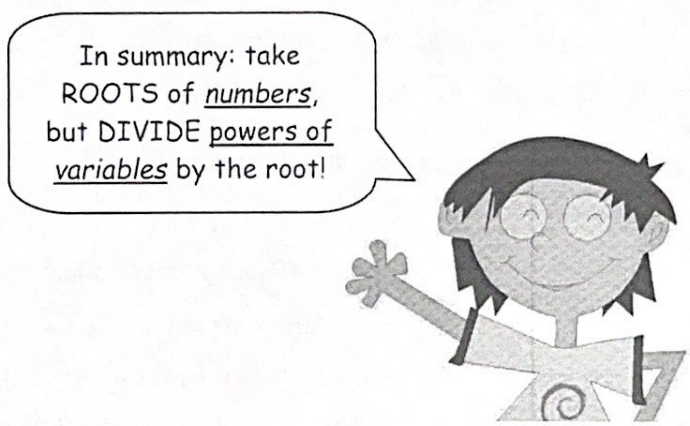
- Example: $\sqrt{x^7} = \sqrt{x^6 \cdot x} = x^3\sqrt{x}$
- Example: $x^2\sqrt{x^9} = x^2\sqrt{x^8 \cdot x} = x^2 \cdot x^4\sqrt{x} = x^6\sqrt{x}$

HOW TO DEAL WITH <u>CUBE ROOTS</u>...

No big deal! Follow the same steps as square roots with these changes:

- Numbers: select the set of factors that has a perfect <u>cube</u>, and take the cube root of that number to put in front of the radical.
  $\sqrt[3]{56} = \sqrt[3]{\_\_ \cdot \_\_} = \_\_\ \sqrt[3]{\_\_}$
- Integers: " peel off " one or two variables, whichever is needed to make the power divisible by <u>three</u>, then divide that power by three to find what to put in front of the radical. $\sqrt[3]{x^7} = \sqrt[3]{\_\_ \cdot \_\_} = \_\_\ \sqrt[3]{\_\_}$
- Example: $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}$
- Example: $\sqrt[3]{x^{17}} = \sqrt[3]{x^{15} \cdot x^2} = x^5\sqrt[3]{x^2}$

Time to practice!



Simplify each radical.

1. $\sqrt{50x^6}$

2. $9\sqrt{147x^9y^4}$

3. $6ac^4\sqrt{300a^{11}b^{12}c^5}$

4. $\sqrt[3]{40x^6}$

5. $-3\sqrt[3]{128a^7b^{14}}$

6. $2x^2\ \sqrt[3]{250x^{10}y^9}$

Rationalizing the Denominator

SINGLE TERM DENOMINATORS

For square roots: Multiply the top AND bottom of the fraction by the radical in the denominator. (And simplify if possible).

$$\frac{3}{\sqrt{6}} \cdot \left(\frac{\sqrt{6}}{\sqrt{6}}\right) = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$$

√ for understanding:
$\sqrt{6}\cdot\sqrt{6} = 6$ **Why?**
Because $\sqrt{6}\cdot\sqrt{6} = \sqrt{36} = 6$

Now *you* rationalize (and simplify) this one:

$$\frac{4}{\sqrt{10}}$$

For cube roots: Multiply the top AND bottom of the fraction by the SQUARE of the radical in the denominator. (And simplify if possible).

$$\frac{3}{\sqrt[3]{6}} \cdot \left(\frac{\sqrt[3]{6^2}}{\sqrt[3]{6^2}}\right) = \frac{3\sqrt[3]{36}}{6} = \frac{\sqrt[3]{36}}{2}$$

√ for understanding:
$\sqrt[3]{6}\cdot\sqrt[3]{6^2} = 6$ **Why?**
Because $\sqrt[3]{6}\cdot\sqrt[3]{6^2} = \sqrt[3]{216} = 6$

Now *you* rationalize (and simplify) this one:

$$\frac{15}{\sqrt[3]{5}}$$

TWO TERM DENOMINATORS

Multiply the top AND bottom of the fraction by the conjugate of the denominator. (And simplify if possible).

$$\frac{5}{\sqrt{6}-4} \cdot \left(\frac{\sqrt{6}+4}{\sqrt{6}+4}\right) = \frac{5(\sqrt{6}+4)}{6-16} = \frac{5(\sqrt{6}+4)}{-10} = \frac{\sqrt{6}+4}{-2}$$

Note: When you multiply conjugates (a-b)(a+b), you get a difference of squares $a^2 - b^2$.

Now *you* rationalize this one:

$$\frac{-3}{5+\sqrt{3}}$$

TIP:
Notice how the simplifying step was done before distributing the 5. This can help you avoid simplifying errors.

TIME TO PRACTICE – Rationalize the denominator. Simplify if possible.

1. $\frac{2}{\sqrt{3}}$

2. $\frac{-6}{\sqrt{7}}$

3. $\frac{10}{\sqrt{6}}$

4. $\frac{-2}{\sqrt{2}}$

5. $\frac{20}{\sqrt{5}}$

6. $\frac{4}{\sqrt[3]{3}}$

7. $\frac{-5}{\sqrt[3]{5}}$

8. $\frac{12}{\sqrt[3]{2}}$

9. $\frac{2}{6+\sqrt{10}}$

→**Careful! In #9, be sure to multiply by the conjugate of the denominator, not just $\sqrt{10}$!**

10. $\frac{-8}{\sqrt{10}-2}$

11. $\frac{12}{\sqrt{5}-\sqrt{6}}$

12. $\frac{3}{\sqrt{15}+5}$

13. $\frac{9}{\sqrt{8}+\sqrt{5}}$

LINEAR EQUATIONS

Horizontal lines are written in the form of y = **a**, where **a** is the number of the y-value of all points on the line. Slope is zero.

Vertical lines are written in the form of x = **a**, where **a** is the number of the x-value of all points on the line. Slope is undefined.

Answer the following questions:

1. A line contains the points (1,5) (4,5) (-2,5). What is the y-value for all these points? ________. Therefore the equation of this line is __________, and the slope is _______________.

2. A line contains the points (6,-13) (6,5) (6,22). What is the x-value for all these points? ________. Therefore the equation of this line is __________, and the slope is _______________.

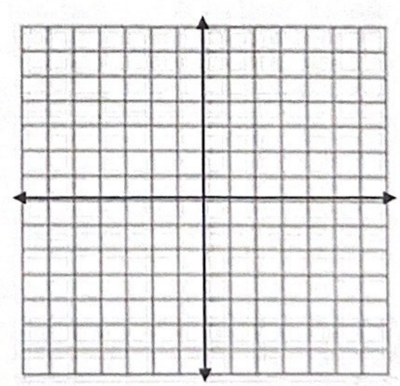
Diagonal lines can be written in the form of y = **mx** + **b** (slope-intercept form), where **m** is the slope of the line, and **b** is the y-intercept.

The slope **m** is the same as $\frac{rise}{run}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

Answer the following questions:

3. A line has a slope of $\frac{1}{3}$ and a y-intercept of (0,2) What is the equation of the line in slope-intercept form? ___________________.

4. Graph the line from #3 on the graph to the right by plotting the point (0,2) and counting up 1, right 3, to plot a second point.
Plot a third point using m = $\frac{1}{3}$ again or by counting down 1, left 3 from (0,2).



5. A line has the points (6,-2) and (8,6) What is the slope of the line? ________.

6. Using the slope from #5 as m, and the point (6,-2) as (x_1, y_1), insert the values in the point-slope equation of a line:

$(y - y_1) = m(x - x_1)$ (y ____) = ____ (x ____)

7. How could you use the two points from #5 to draw the line?

8. How could you use the point and the slope in #6 to draw the line?

9. Using the point-slope equation of a line you wrote in #6, convert that into slope-intercept form $y = mx + b$ by distributing m and then isolating y.

10. If your graph paper domain and range is -10 to 10, why is using the point-slope form of this equation (#9) not the most logical method of graphing?

11. Find the slope for the line going through (8,3) (5, 7).

12. Write a point-slope equation for #11.

13. Convert the equation from #12 into slope-intercept form.

Solving Equations

<u>Directions</u>: *Solve for x, noting the key step that is required for each section.*

Key step: Multiply

(use cross multiplication)

1. $3(x + 3) - 6 = 5x - 7$
2. $(x - 4)^2 - 11 = x^2 - 12x + 21$
3. $\frac{2x + 3}{5x - 2} = \frac{4}{5}$

Key step: Take square root of both sides

4. $x^2 = 100$
5. $(x - 10)^2 = 16$
6. $(6x - 2)^2 = 20$

Key step: Use quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

7. $2x^2 + x - 3 = 0$
8. $x^2 + 9x + 15 = 0$
9. $10x^2 - 2x = 5$

Key step: Square (or cube) both sides (Hint: isolate radical first)

10. $\sqrt{x - 12} = 5$
11. $\sqrt{2x + 2} - 4 = 0$
12. $-3 + \sqrt[3]{x - 8} = 0$

Key step: Factor

13. $10x^2 + 20x = 0$
14. $x^2 - 49 = 0$
15. $x^2 + 12x + 32 = 0$

Long Division – can be used when dividing any polynomials.

Synthetic Division—can ONLY be used when dividing a polynomial by a linear polynomial.

EX: $\dfrac{2x^3+3x^2-6x+10}{x+3}$

Long Division

$$\frac{2x^3+3x^2-6x+10}{x+3}$$

$$2x^2-3x+3+\frac{1}{x+3}$$

$$x+3\overline{\big)\,2x^3+3x^2-6x+10}$$

$$(-)\left(2x^3+6x^2\right)$$

$$-3x^2-6x$$

$$(-)\left(-3x^2-9x\right)$$

$$3x+10$$

$$(-)\ (3x+9)$$

$$1$$

Synthetic Division

$$\frac{2x^3+3x^2-6x+10}{x+3}$$

$$\frac{2x^3+3x^2-6x+10}{x+3}$$

-3	2	3	-6	10
		-6	9	-9
	2	-3	3	1

$$=2x-3x+3+\frac{1}{x+3}$$

Divide each polynomial using long division OR synthetic division.

$\dfrac{c^3-3c^2+18c-16}{c^2+3c-2}$

$\dfrac{x^4-2x^2-x+2}{x+2}$

Formulas That You Are Expected To Know

Linear

Slope between two points: $m=\frac{y_2-y_1}{x_2-x_1}$

Point -Slope Form: $y-y_1=m(x-x_1)$

Slope-Intercept Form: $y=mx+b$

Standard Form: $Ax+By=C$ (A, B, and C must be integers)

Horizontal Line: $y=k$ (zero slope)

Vertical Line: $x=k$ (no slope)

Distance Formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Midpoint Formula: $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Quadratic

Quadratic Formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Discriminant: b^2-4ac

$b^2-4ac>0$	2 real unequal roots
$b^2-4ac=0$	1 double real root
$b^2-4ac<0$	2 complex conjugate roots

Standard Form of a Parabola: $y=ax^2+bx+c$ or $x=ay^2+by+c$

Vertex Form of a Parabola: $y=a(x-h)^2+k$ or $x=a(y-k)^2+h$

$a=\frac{1}{4p}$; p is the distance from vertex to focus or p is the distance from vertex to directrix

Solving different types of equations:

Quadratic equation - factor, complete the square, quadratic formula

Quadratic-like equation - use a dummy variable

Rational equation - remember to check solutions in original equation for any restrictions on the domain

Radical equation - remember to check solutions for possible extraneous roots

Exponential and Logarithmic Functions

Exponential Function: $y=b^x$ with $b>0,\neq 1$; $Domain:\Re$ $Range:\Re>0$

Logarithmic Function: $y=\log_b x$ with $b>0,\neq 1$; $Domain:\Re>0$ $Range:\Re$

Exponential/Logarithmic Identities and Properties

$\log_b b = 1$	$\ln e = 1$	$\log_b x + \log_b y = \log_b xy$
$\log_b 1 = 0$	$\ln 1 = 0$	$\log_b x - \log_b y = \log_b \frac{x}{y}$
$\log_b b^x = x$	$\ln e^x = x$	
$b^{\log_b x} = x$	$e^{\ln x} = x$	$k \log_b x = \log_b x^k$

Solving exponential/logarithmic equations

$b^x = b^y ...iff...x = y$ $\qquad$ $\log_b x = \log_b y ...iff...x = y$

Converting from exponential to logarithmic form

$x = b^y \Leftrightarrow y = \log_b x$

Change of Base Formula: $\log_b a = \frac{\log_c a}{\log_c b}$ or $\log_b a = \frac{\log a}{\log b}$ or $\log_b a = \frac{\ln a}{\ln b}$

Exponential Application Formulas: $A(t) = A_0(1 \pm r)^t$ $\qquad$ $A(t) = A_0 e^{kt}$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ $\qquad$ $A = Pe^{rt}$

<u>Equations of a Circle</u>: $x^2 + y^2 = r^2$; center $= (0,0)$ and radius $= r$

$(x-h)^2 + (y-k)^2 = r^2$; center $= (h,k)$ and radius $= r$

$x^2 + y^2 + ax + by + c = 0$; must complete the square in x and y